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### Estimating the gravity model without gravity using panel data

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# Estimating the gravity model without gravity using panel data\*

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## Abstract

This paper examines the effects of zero trade on the estimation of the gravity model using both simulated and real data with a panel structure, which is different from the more conventional cross-sectional structure. We begin by showing that the usual log-linear estimation method can result in highly deceptive inference when some observations are zero. As an alternative approach, we suggest using the poisson fixed effects estimator. This approach eliminates the problems of zero trade, controls for heterogeneity across countries, and is shown to perform well in small samples.

**JEL Classification:** F10; F15; C15; C23.

**Keywords:** Gravity model of trade; Poisson regression model; Panel data; Monte Carlo simulation.

## 1 Introduction

The gravity model of trade has been widely used to estimate the impact of various policy issues, including preferential trade agreements, currency unions, and border effects. The model has a long tradition in social sciences where it has been used to model, for example, migration. In economics, the model has become very popular due to its success in explaining trade

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flows among countries. Some critique for the lack of theoretical underpinnings has emerged but much progress has been made and now the gravity model rests on a solid theoretical foundation. Instead, the focus has shifted towards the estimation techniques used.

The gravity model has traditionally been estimated using cross-sectional data. However, this has been shown to generate biased results since heterogeneity among the countries is typically not controlled for in an appropriate way, see Cheng and Wall (2005), and Cheng and Tsai (2008). To mitigate this problem, researchers have turned towards panel data, which have the advantage that they permit more general types of heterogeneity. For example, consider estimating the impact of currency unions on trade while controlling for country-pair propensity to trade. For a single cross-section, these controls can only depend on observed country-pair attributes such as common language, and estimates can thus be biased if there is additionally an unobserved component to the propensity to trade. With panel data, such unobserved heterogeneity can be readily controlled for by means of a country-pair fixed effects model, which is more general than both the pooled cross-sectional and country specific fixed effects panel data models.

The single most popular approach to estimating the gravity model using panel data is to first make it linear by taking logarithms and then to estimate the resulting log-linear model by the fixed effects least squares (LS). However, although simple to implement, this approach is problematic because the log-linearized model is not defined for observations with zero trade. Moreover, even though the proportion of observations with zero trade may vary somewhat depending on, among other things, the size of the sample, it is usually quite significant, suggesting that the proper handling of these zeros is potentially very important. Another problem is that the LS estimator of the log-linearized model may be both biased and inefficient in the presence of heteroskedasticity.

Two of the most common approaches to handle the presence of zero trade are to either simply discarding the zeros from the sample, or to add a constant factor to each observation on the dependent variable. The first strategy is correct as long as the zeros are randomly distributed. However, if the zeros are not random, as is usually the case, then this induces a selection bias. This problem is often ignored in applied work, but could be handled by using sample selection correction. In a recent contribution, Helpman *et al.* (2008) propose a theoretical model rationalizing the zero trade flows and suggest estimating the gravity equation with a correction for the probability of countries to trade. To estimate the model

they apply a two-step estimation technique similar to sample selection models. However, in order to implement the new estimator, the researcher needs to find a suitable exclusion restriction for identification of the second stage equation, which can be quite difficult. The problem with bias and inefficiency in the presence of heteroskedasticity has been largely ignored by applied researchers.

In this paper, we explore and extend upon an idea first pointed out by Wooldridge (2002), namely that the fixed effects panel poisson maximum likelihood (ML) estimator can be applied also to continuous variables. We therefore propose estimating the gravity model directly from its non-linear form by using the poisson ML estimator. Since this removes the need to linearize the model by taking logarithms, the problem with zero trade disappears. A similar approach has recently been proposed by Silva and Tenreyro (2006), who also use the poisson ML estimator. However, they use cross-sectional data, and focus mainly on the issue of heteroskedasticity. Our approach is more general in the sense that it permits one to get rid of the problems of zero trade and heteroskedasticity while simultaneously taking care of the bias caused by country specific heterogeneity, which cannot be accomplished when using cross-sectional data.

Our simulation results suggest that the new estimation method is superior to the conventional approach of applying LS to the log-linearized model. In particular, it is shown that the conventional approach is likely to result in severe bias and misleading inference even if the fraction of observations with zero trade is very small. On the other hand, the poisson ML estimator generally performs very well with only small bias and size distortion. Therefore, since the poisson ML estimator is becoming increasingly available using standard statistical software packages, these results suggest that it should be a valuable tool for econometric analysis of the gravity model. As an empirical illustration, we consider the trade effects of the 1995 European Union (EU) enlargement.

The remainder of this paper is organized as follows. Section 2 briefly outlines the gravity model and the problems of zero trade. Section 3 then presents the Monte Carlo simulations, while Section 4 contains the application. Section 5 concludes.

## 2 The problem of zero gravity

Let  $M_{ijt}$  denote the bilateral trade between countries  $i = 1, \dots, n$  and  $j = 1, \dots, n$  with  $i \neq j$  at time  $t = 1, \dots, T$ , as measured by the imports of country  $i$  from country  $j$ . For convenience,

the total number of observations per time period, which is given by  $n(n-1)$ , is henceforth denoted by  $N$ .<sup>1</sup> A common empirical formulation of the gravity model for bilateral trade includes the GDP levels of the two countries,  $Y_{it}$  and  $Y_{jt}$  say, as well as  $D_{ijt}$ , a dummy variable representing for example some contiguity, common language or free-trade agreement effect. This formulation of the gravity equation can be written algebraically as

$$\lambda_{ijt} = E(M_{ijt}|Y_{it}, Y_{jt}, D_{ijt}) = \exp(\gamma D_{ijt}) Y_{it}^{\beta_1} Y_{jt}^{\beta_2}. \quad (1)$$

Because only a very limited amount of heterogeneity between the country pairs is allowed in the parametrization of the regression function, conventional cross-section estimates of the gravity model are generally biased. With panel data, on the other hand, we can easily permit for such heterogeneity by means of  $N$  country-pair specific effects, denoted  $\alpha_{ij}$ . These effects may be different depending on the direction of trade and enters (1) multiplicatively in the following fashion

$$E(M_{ijt}|Y_{it}, Y_{jt}, D_{ijt}, \alpha_{ij}) = \exp(\alpha_{ij} + \gamma D_{ijt}) Y_{it}^{\beta_1} Y_{jt}^{\beta_2} = \exp(\alpha_{ij}) \lambda_{ijt}.$$

This implicitly defines the following regression

$$M_{ijt} = \exp(\alpha_{ij}) \lambda_{ijt} + e_{ijt},$$

which can be written equivalently as

$$M_{ijt} = \exp(\alpha_{ij}) \lambda_{ijt} v_{ijt}, \quad (2)$$

where  $e_{ijt}$  is a mean zero disturbance that is independent of the regressors, and where  $v_{ijt} = 1 + e_{ijt}/\exp(\alpha_{ij})\lambda_{ijt}$  is a heteroskedastic disturbance term with  $E(v_{ijt}|Y_{it}, Y_{jt}, D_{ijt}, \alpha_{ij}) = 1$ . Moreover, since  $\alpha_{ij}$  will generally be correlated with the explanatory variables, random effects estimation of (2) will be inconsistent. To circumvent this, it is common to treat  $\alpha_{ij}$  as fixed.

Suppose for a moment that  $M_{ijt}$  is strictly positive. One of the most common approaches to estimate the regression in (2) is to first make it linear by taking logarithms, which yields

$$\ln(M_{ijt}) = \alpha_{ij} + \ln(\lambda_{ijt}) + \ln(v_{ijt}) = \alpha_{ij} + \gamma D_{ijt} + \beta_1 \ln(Y_{it}) + \beta_2 \ln(Y_{jt}) + \ln(v_{ijt}). \quad (3)$$

Since the model is now linear, it is readily estimable using LS. However, this is only possible as long as  $M_{ijt}$  is nonzero, which is not always the case. Indeed, a common feature of trade

<sup>1</sup>Note that since each country is both an exporter and an importer in a bilateral trade relation, each country pair is observed twice. The number of observations is therefore twice the number of country pairs.

data is that the bilateral trade can sometimes be zero. Although this poses no problem when estimating the gravity model based on its multiplicative form in (2), as the logarithm is defined only for positive outcomes, the log-linear regression in (3) is no longer admissible. A common solution to this problem is to drop all observations with zero trade, and then to estimate (3) based on the resulting truncated sample. However, although this approach certainly eliminates the zeros, it simultaneously induces a bias to the LS estimator, which is why truncating the sample should be avoided as a matter of practice.

A natural alternative approach in situations such as this, when the model cannot be log-linearized, is to estimate it from its multiplicative form directly. In so doing, note that the fixed effects conditional mean can be written as

$$\lambda_{ijt} = \exp(\alpha_{ij} + \gamma D_{ijt} + \beta_1 \ln(Y_{it}) + \beta_2 \ln(Y_{jt})), \quad (4)$$

which is known as the exponential regression function. This regression follows naturally from the multiplicative form of (1) and ensures that  $\lambda_{ijt}$  is nonnegative, which is very convenient as trade cannot be negative. Thus, the conventional additive regression in (3) is likely to be unsatisfactory here as it cannot ensure the nonnegativity of trade.

The estimation of (4) has been studied by Hausman *et al.* (1984), who consider the special case when the data are measured in nonnegative integers. They propose using a version of the conventional poisson ML estimator, which is modified to account for the fixed effects. In so doing, the authors eliminate the fixed effects by conditioning on  $\sum_{t=1}^T M_{ijt}$ , a sufficient statistic for  $\alpha_{ij}$ , which in our case yields the following log-likelihood function

$$\ln(L) = \sum_{i \neq j}^n \sum_{t=1}^T \Gamma(M_{ijt} + 1) - \sum_{i \neq j}^n \sum_{t=1}^T M_{ijt} \ln \left( \sum_{s=1}^T \frac{\lambda_{ijs}}{\lambda_{ijt}} \right),$$

where  $\Gamma$  is the gamma function. As noted by the authors, given that the regression in (4) is correctly specified, consistency of the resulting fixed effects poisson ML slope estimator follows directly by standard ML theory, see for example Gourieroux *et al.* (1984).<sup>2</sup> The Hausman *et al.* (1984) poisson conditional ML estimator is the same as the poisson ML estimator

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<sup>2</sup>As long as (4) holds the poisson estimator works, see for example Wooldridge (2002) and Winkelmann (2008). In fact, neither (4) nor the maximization of the log-likelihood function require that the dependent variable is a count. It could be a binary variable or, as in our case, a nonnegative continuous variable. This property of the estimator has been used by Silva and Tenreyro (2006). The interpretation of the estimated coefficients is similar to the interpretation of the coefficients in the log-linear model. That is, the estimated coefficient reflects the elasticity of the dependent variable with respect to the relevant independent variable. In the case of an dummy variable, the estimated coefficient provides a reasonable approximation for small estimated values, see Winkelmann (2008) for a more elaborative discussion.



in a model with individual specific constants, which in turn is equivalent to the moment estimator in a model where the fixed effects are replaced by  $\frac{1}{T} \sum_{t=1}^T M_{ijt} / \frac{1}{T} \sum_{t=1}^T \lambda_{ijt}$ , the ratio of within group means. Alternative estimators of the fixed effects poisson model include the quasi-differenced generalized method of moments estimator and the pre-sample mean estimator that replaces the fixed effects by the pre-sample mean of the dependent variable, see for example Blundell *et al.* (2002) for a detailed discussion.<sup>3</sup>

Having estimated the slopes, an estimate of the fixed effects can be obtained by simply replacing  $\lambda_{ijt}$  in  $\sum_{t=1}^T M_{ijt} / \sum_{t=1}^T \lambda_{ijt}$  by its ML estimate. Note that this gives an estimate of  $\exp(\alpha_{ij})$ , not of  $\alpha_{ij}$ , which is unidentified in the fixed effects formulation of the model. In order to identify  $\alpha_{ij}$ , a random effects assumption is needed. But such assumptions are generally not satisfied in practice, and so we only consider the fixed effects specification.

Although the poisson ML estimator is consistent, valid inference requires the correct specification of both the conditional mean and variance, which necessitates that

$$\lambda_{ijt} = \text{var}(M_{ijt} | Y_{it}, Y_{jt}, D_{ijt}). \quad (5)$$

However, note that the validity of (4) and (5) does not require the data to be poisson distributed. In fact,  $M_{ijt}$  does not have to be an integer at all. This suggests that we can use the fixed effects poisson ML to estimate the gravity model. Since this estimator does not require  $M_{ijt}$  to be nonzero, it is expected to produce better results than LS in panels where some trade flows are zero. Moreover, if it is consistency that we are interested in, then (5) does not have to hold either, so the data do not have to be equidispersed. In the next section, we elaborate on this point.

### 3 Monte Carlo study

In this section, we investigate the small-sample properties of the LS and ML estimators in the presence of zero observations through Monte Carlo simulations. The data generating process used for this purpose is given by

$$M_{ijt} = \exp(\alpha_{ij} + \gamma D_{ijt} + \beta Y_{ijt}) v_{ijt}, \quad (6)$$

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<sup>3</sup>Another possibility is to use the zero inflated poisson (ZIP) model. But so far it seems that the estimation of this model with fixed effects has not yet been analyzed in the literature. In fact, Winkelmann (2008) points out that the properties of the fixed effects poisson ML estimator does not carry over to the ZIP model, and that the estimation of this model is still an open issue.



where  $\alpha_{ij} = \gamma = \beta = 1$  for simplicity. Since  $Y_{ijt}$  is usually positive in applied work, we set  $Y_{ijt} \sim U(0, 1)$ . Moreover, if we let  $\tau_{ij} \sim U(0, 1)$  denote the location of the break, then the dummy variable  $D_{ijt}$ , representing for example a preferential trade agreement, is such that  $D_{ijt} = 1$  if  $t > \tau_{ij}T$  and zero otherwise.

The disturbance  $v_{ijt}$  is key in this data generating process. In particular, it is assumed that  $v_{ijt}$  is a log-normally distributed variable with mean one and variance  $\sigma_{ij}^2$ . We have two variance cases. In the case 1,  $\sigma_{ij}^2 = 1$ , which implies that

$$\text{var}(M_{ijt}|Y_{ijt}, D_{ijt}) = \exp(\alpha_{ij} + \gamma D_{ijt} + \beta Y_{ijt})^2,$$

while in case 2,  $\sigma_{ij}^2 = 1/\exp(\alpha_{ij} + \gamma D_{ijt} + \beta Y_{ijt})$  so that

$$\text{var}(M_{ijt}|Y_{ijt}, D_{ijt}) = \exp(\alpha_{ij} + \gamma D_{ijt} + \beta Y_{ijt}).$$

Thus, we expect the LS estimator to perform relatively well in case 1, while we expect the poisson ML estimator to perform relatively well in case 2, as condition (5) is now satisfied.<sup>4</sup> In both cases, we generate data by drawing 1,000 panels, each consisting of  $N$  observations on each of the  $T$  time series.

The results are organized according to the two cases described above. In each case, we want to examine the effect of zero observations in the data. Both the LS and poisson ML estimators are considered.<sup>5</sup> The former is implemented using both truncated data and  $\ln(M_{ijt} + 1)$  as dependent variable. However, note that since  $M_{ijt} > 0$  in this data generating process, the log-linear model is no longer inadmissible. Hence, to be able to study the effect of truncating the sample we use a positive truncation threshold parameter, which is such that the fraction of truncated observations is exactly  $\delta$ . For brevity, we only report the mean bias and the size of a nominal 5% level  $t$ -test of the null hypothesis that the parameter of interest is equal to its true value versus the alternative that it is not.<sup>6</sup>

Besides the LS and poisson ML estimators, we also experimented with the negative binomial ML estimator of Hausman *et al.* (1984), which relaxes condition (5). But since the

<sup>4</sup>Other values of  $\sigma_{ij}^2$  produced very similar results and are thus not reported.

<sup>5</sup>The poisson ML estimator is implemented using the GAUSS optimization library OPTMUM. We use the BFGS gradient algorithm with numerical derivatives. The standard errors of the estimated parameters are computed based on the conventional Hessian method, which generally worked best in the simulations. The truncated LS is used to start up the estimation.

<sup>6</sup>We also simulated the power of the  $t$ -tests. However, since the size of the LS based tests turned out to be heavily distorted, with rejection frequencies close to 100% in most experiments, power is not very interesting, and the results are therefore not reported.

performance was so unsatisfactory, the results are not included here but are available from the corresponding author upon request. The panel version of the quasi-ML estimator discussed in *Gourieroux et al.* (1984) also performed very poorly, and was therefore removed.<sup>7</sup> Another possibility is to treat the zeros as a sample selection issue, and to estimate the model using an estimator that eliminates the selectivity bias. We tried the Kyriazidou (1997) estimator, which is a popular two-step procedure to difference out both the bias and fixed effects. However, as with the negative binomial and quasi-ML estimators, the results from this estimator were very poor, and were therefore removed.<sup>8</sup>

The results reported in Table 1 for the LS and poisson ML estimators can be summarized as follows. First, as expected, LS estimation with  $\ln(M_{ijt} + 1)$  as the dependent variable generally produces very poor results. In particular, it is seen that the estimators of  $\gamma$  and  $\beta$  both suffer from substantial downwards bias, which do not show any tendency to vanish as the sample size increases. Moreover, the results of the size of the  $t$ -tests suggest that inference based on this estimation method is likely to be highly deceptive. In fact, with this method, we always end up rejecting the null hypothesis. Thus, based on these results, we recommend not using LS estimation based on  $\ln(M_{ijt} + 1)$ .

Second, the results on the truncated LS estimator are mixed. At one end of the scale, we have case 1 when there is no truncation, in which the performance, both in terms of bias and size accuracy, is very good. At the other end, we have the case when  $\delta > 0$ , in which Table 1 shows that the performance is poor, and that the problems with bias and size distortion are highly potent, even for a truncation as small as 10%. Apparently, the truncation makes the LS estimator both downwards biased and unfit for inference. Thus, from an empirical point of view, it seems highly unlikely that the truncated LS is able to deliver any meaningful results at all.

In addition to the problems associated with truncating the data, Table 1 points to another important shortcoming with the truncated LS estimator. In particular, it seems as that the heteroskedasticity in case 2 induces both severe size distortions as well as a sizeable bias that persists even in large panels.

<sup>7</sup>The quasi-ML estimator only requires that the conditional mean in (4) is correctly specified, and does not make use of (5), see for example *Gourieroux et al.* (1984) and *Wooldridge* (2002).

<sup>8</sup>We used the  $T = 2$  version of the Kyriazidou (1997) estimator, which is relatively easy to compute, but preliminary results suggest that the poor performance extends also to the case when  $T > 2$ . Also, for this experiment, the data generating process was adapted so as to fit the sample selection setting of Kyriazidou (1997).

Although this may appear somewhat counterintuitive at first, as pointed out by Silva and Tenreyro (2006), it is actually a direct consequence of the well-known Jensen inequality. To appreciate this, consider the data generating process in (6) where  $E(v_{ijt}|Y_{ijt}, D_{ijt}) = 1$ . The LS estimator of the parameters in the log-linear model (3) are consistent only if  $E(\ln(v_{ijt})|Y_{ijt}, D_{ijt}) = 0$ . However, although  $\ln(E(v_{ijt}|Y_{ijt}, D_{ijt})) = 0$ , by the Jensen equality,  $E(\ln(v_{ijt})|Y_{ijt}, D_{ijt}) \neq 0$ . Indeed, since  $E(v_{ijt}|Y_{ijt}, D_{ijt})^2 = 1$  in our case, by using the properties of the log-normal distribution, we have that

$$E(\ln(v_{ijt})|Y_{ijt}, D_{ijt}) = \ln\left(\frac{1}{1 + \sigma_{ij}^2}\right),$$

which is not equal to zero unless of course  $\sigma_{ij}^2$  is zero too. As a result, the LS estimator in (3) will generally be biased.

Third, except possibly for case 1 when there is no truncation, the results show that the poisson ML consistently outperforms the other estimators in terms of bias. In fact, by looking at Table 1, it would appear as that the bias is practically nonexistent even for as small panels as  $T = 10$  and  $N = 500$ , which correspond approximately to 10 time series observations for 23 countries. We also see that the size is very close to the nominal 5% level in case 2 but that it is distorted in case 1, which is partly expected since condition (5) is not satisfied in this case.

One possibility to get rid of the distorted standard errors of the ML estimator is to use the bootstrap. This approach has become very popular in applied work, and it will therefore be used in this paper. The particular algorithm used is taken from Cameron and Trivedi (1998), who make a very simple proposal, in which the dependent and independent variables are resampled in pairs.<sup>9</sup> Some simulations of the resulting bootstrapped  $t$ -statistic based on 100 bootstrap replications are reported in Table 2. As expected, we see that the size of the bootstrapped test generally lies much closer to the 5% level than the size of the asymptotic test. Also, the  $t$ -statistics appear to be well centered around zero.

In summary, we find that the poisson ML show smaller bias than the two LS estimators considered and, at the same time, maintain relatively good size properties in small samples. Since the poisson ML with bootstrapped standard errors is now readily available through existing software packages such as STATA, it should be considered a feasible alternative to estimation by LS.

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<sup>9</sup>Another possibility is to use the wild bootstrap, see Cameron and Trivedi (1998) for a discussion.

## 4 An application to the 1995 EU enlargement

We have shown that log-linear LS estimation of the gravity model yields biased results. In this section, we demonstrate these findings by estimating the trade effects of the adhesion of Austria, Finland and Sweden to the EU in 1995. The sample that we use for this purpose cover the period 1992 to 2002 and consists of import data for EU and other developed countries from all trade partners except oil exporting countries and formerly planned economies in Central and Eastern Europe, as defined in Direction of Trade Statistics (International Monetary Fund, 2005). The GDP and population data comes from World Development Indicators (World Bank, 2005).

The estimated gravity equation can be written as

$$M_{ijt} = \exp(\alpha_{ij} + \mu_t + \gamma_1 D_{it} + \gamma_2 D_{jt} + \gamma_3 D_{ijt}) Y_{it}^{\beta_1} Y_{jt}^{\beta_2} N_{it}^{\beta_3} N_{jt}^{\beta_4} v_{ijt}, \quad (7)$$

or equivalently in its log-linear form

$$\begin{aligned} \ln(M_{ijt}) &= \alpha_{ij} + \mu_t + \gamma_1 D_{it} + \gamma_2 D_{jt} + \gamma_3 D_{ijt} + \beta_1 \ln(Y_{it}) + \beta_2 \ln(Y_{jt}) \\ &+ \beta_3 \ln(N_{it}) + \beta_4 \ln(N_{jt}) + \ln(v_{ijt}), \end{aligned} \quad (8)$$

where  $M_{ijt}$  denotes the nominal imports of country  $i$  from country  $j$ ,  $Y_{it}$  and  $Y_{jt}$  denote the real GDP of the two countries, and  $N_{it}$  and  $N_{jt}$  denote their population. The fixed effects  $\alpha_{ij}$  capture all types of unobserved country-pair specific heterogeneity that is constant over time, while the time effects  $\mu_t$  capture all forms of time-varying heterogeneity that is shared among the country pairs.

The dummy variables  $D_{it}$ ,  $D_{jt}$  and  $D_{ijt}$  are key in this model. The variable  $D_{it}$  equals one if country  $i$  is a member of the EU at time  $t$  while country  $j$  belongs to the rest of the world. The second dummy variable  $D_{jt}$  equals one if country  $j$  is a member of the EU while  $i$  belongs to the rest of the world. Similarly,  $D_{ijt}$  equals one if both  $i$  and  $j$  are members of the EU at time  $t$ . In other words, the three dummy variables take the value one for EU imports from the rest of the world, EU exports to the rest of the world and intra-EU trade, respectively.

The rest of the world is defined as all countries in the sample that are not members of the EU at any given time in the sample. This enables us to identify the effect of the enlargement on the trade of new EU members as opposed to the effect of changes in the size of the rest of the world. To appreciate this, note that if the rest of the world also included new members,

the dummy variable  $D_{it}$  would capture not only the import effect on the new members but also the effect of the change in the composition of the rest of the world, as the imports from the new members to the old ones would no longer be classified as imports from the rest of the world. A similar argument applies to the construction of  $D_{jt}$ .

A consequence of this definition of the rest of the world is that, since fixed effects absorb all heterogeneity that is constant over time, the trade effect for countries that have been members of the EU for the whole sample period cannot be identified. Thus, the dummy variables capture only the effect on countries that have changed their EU status at least one time. That is, the dummy variables capture the effect of the Austrian, Finnish and Swedish accession to the EU. Specifically,  $\gamma_1$  measures the trade diversion or changes in EU imports from the rest of the world. Similarly,  $\gamma_2$  measures the effect on EU exports to the rest of the world, sometimes called export diversion. Finally,  $\gamma_3$  measures trade creation, resulting from the increased intra-EU trade following the enlargement.

Economic integration should increase trade between countries integrating. Thus, we expect the trade creation, as measured by  $\gamma_3$ , to be positive. This effect can be separated into pure trade creation, or increased trade due to lower prices on imports from the other countries in the EU, and trade diversion, which implies a shift in imports from more efficient producers in the rest of the world to less efficient producers within the EU. A negative sign on  $\gamma_1$  would thus indicate trade diversion. Similarly, export diversion occurs if exports to the rest of the world decreases as a result of the integration process, but exports could also increase. The expected sign of  $\gamma_2$  is therefore ambiguous.

The empirical results are contained in Table 3. It is seen that the enlargement of the EU induced significant trade diversion but no trade creation. This absence of trade creation is, however, not surprising since the new members were part of a free trade area with the EU prior to the membership. When joining the EU, the new members implemented the Common External Tariff, which changed the tariffs on their imports from the rest of the world. Note that the trade diversion effect is rather large in comparison to the trade creation effect. Although counterintuitive at first, one should keep in mind that several countries with preferential access to the EU market, such as those that joined the EU in 2004, have been excluded from our sample, so trade might have been diverted away from suppliers on the world market to suppliers with preferential access to the EU market. Moreover, taken as a fraction of total trade, the diversion effect is probably quite small since the estimation results

only capture the effect on imports to Austria, Sweden and Finland and not changes in the total imports of the EU.

Even though the number of zeros is comparatively small in our sample, only 10%, when comparing the results obtained from the various estimators, we see that the difference can be substantial. In particular, for the GDP and population variables, the poisson ML estimates are typically larger than their LS counterparts. This finding is well in line with the Monte Carlo evidence suggesting that both LS estimators are downwards biased. Moreover, while the truncated LS estimator indicates that changes in GDP of importing countries does not effect imports, the ML estimator gives a more plausible estimate close to unity.

It should also be mentioned that the LS estimates of the GDP and population parameters appear to be rather unstable, and to a large extent dependent on the time period used, which is probably due to the fact that these variables seem to be quite highly correlated. On the other hand, the corresponding LS estimates of the effects of trade liberalization appear to be very robust, and show almost no variation between time periods. Similarly, all ML estimates seem vary robust to changes in the time period.

For the dummy variables, the differences are less marked. In particular, although the sign and significance of the estimates do not differ much, the magnitude of the estimates varies quite substantially. The LS estimator indicates that the trade diversion is twice as large as implied by the ML estimator and, while the LS estimate of the trade creation effect is slightly negative, it is positive for the ML estimator.

In summary, the results presented in this section highlight the importance of using appropriate estimation techniques to be able to draw correct inference.

## 5 Conclusions

The gravity model has become a standard tool for evaluating policies affecting trade and it is widely used to assess the effects of preferential trade agreements and currency unions or to calculate trade potential, among other things. It is well known that the gravity model should be estimated by panel data to mitigate the bias due to failure to fully control for country heterogeneity. A very popular way to accomplish this is to first linearize the model by taking logarithms and then to apply the conventional fixed effects LS estimator.

In this paper, we argue that this approach is likely to be very misleading with severely biased estimates and  $t$ -statistics. There are two reasons for this. Firstly, since trade cannot

be zero in the log-linearized model, all zeros must either be discarded or replaced by some arbitrary positive value, which induces a sample selection bias. Secondly, the heteroskedasticity inherent in the log-linear formulation of the gravity model can render the LS estimates both biased and inefficient. By contrast, being based on the gravity model in its original non-linear form, the fixed effects poisson ML estimator does not suffer from these weaknesses and is therefore expected to yield more accurate results.

Our assertion is verified by means of Monte Carlo simulations and illustrated via an application to the 1995 EU enlargement. The simulations show that the performance of the log-linear approach is likely to be so poor that it may not even be meaningful to interpret the results. On the other hand, the poisson ML estimator performs well with only a very small bias and good size accuracy in most cases. Still, in some data generating processes, the results show that the estimated standard errors can be downward biased. To alleviate this, we suggest using bootstrapped standard errors. The empirical application points to a significant difference between the estimators with respect to both the main explanatory variables and the trade effects of the 1995 EU enlargement, thus underlining the importance of using the proper estimation technique.

To conclude, we recommend not estimating the gravity model from its log-linear form. Instead, we propose estimating the model directly from its non-linear form using the fixed effects poisson ML estimator with bootstrapped standard error. Our proposal provide researchers with a simple framework for analyzing the gravity model while at the same time avoiding potential bias due to zero trade. This, together with the fact that the poisson ML estimator can now be implemented using many standard statistical software packages such as STATA, makes our proposal definitely seem worthwhile.



Table 1: Simulated bias and tests size for the ML and LS estimators.

$\delta$	Case	$N$	$T$	Mean bias						Size of the $t$ -test at the 5% level					
				$\hat{\gamma}_{ls}^a$	$\hat{\beta}_{ls}^a$	$\hat{\gamma}_{ls}^b$	$\hat{\beta}_{ls}^b$	$\hat{\gamma}_{ml}$	$\hat{\beta}_{ml}$	$\hat{\gamma}_{ls}^a$	$\hat{\beta}_{ls}^a$	$\hat{\gamma}_{ls}^b$	$\hat{\beta}_{ls}^b$	$\hat{\gamma}_{ml}$	$\hat{\beta}_{ml}$
0	1	500	10	0.1	-0.1	-36.4	-36.9	0.1	-0.1	7.3	5.8	100.0	100.0	24.6	31.8
		1000	10	0.0	0.0	-36.5	-36.8	-0.1	-0.1	6.6	5.8	100.0	100.0	25.6	32.2
		500	20	0.1	0.1	-36.5	-36.7	0.1	0.2	6.2	6.1	100.0	100.0	24.1	33.4
		1000	20	-0.1	0.0	-36.5	-36.8	-0.1	-0.2	5.9	6.2	100.0	100.0	26.5	31.9
	2	500	10	13.8	14.1	-25.8	-26.6	-0.1	0.0	100.0	99.8	100.0	100.0	5.0	6.3
		1000	10	13.8	14.2	-25.8	-26.5	0.1	0.1	100.0	100.0	100.0	100.0	3.7	4.4
		500	20	13.8	14.1	-25.8	-26.6	0.0	0.0	100.0	100.0	100.0	100.0	6.1	4.8
		1000	20	13.9	14.2	-25.8	-26.6	0.1	0.1	100.0	100.0	100.0	100.0	4.9	5.3
0.1	1	500	10	-21.8	-21.8	-36.5	-36.8	-0.1	-0.1	100.0	99.9	100.0	100.0	25.3	29.6
		1000	10	-21.9	-21.8	-36.6	-36.8	-0.1	-0.1	100.0	100.0	100.0	100.0	23.9	29.5
		500	20	-21.9	-21.9	-36.6	-36.8	-0.1	-0.1	100.0	100.0	100.0	100.0	27.1	31.3
		1000	20	-21.8	-21.9	-36.5	-36.8	0.0	0.0	100.0	100.0	100.0	100.0	25.3	32.2
	2	500	10	-9.4	-13.4	-25.7	-26.6	0.1	-0.1	99.8	99.8	100.0	100.0	4.7	4.9
		1000	10	-9.5	-13.5	-25.8	-26.6	0.0	-0.1	100.0	100.0	100.0	100.0	5.5	4.9
		500	20	-9.4	-13.4	-25.8	-26.6	0.0	-0.1	100.0	100.0	100.0	100.0	5.6	4.5
		1000	20	-9.5	-13.3	-25.8	-26.5	0.0	0.1	100.0	100.0	100.0	100.0	5.7	5.7
0.3	1	500	10	-44.8	-40.9	-36.5	-36.8	-0.1	-0.2	100.0	100.0	100.0	100.0	27.2	29.5
		1000	10	-44.7	-41.0	-36.5	-36.8	0.0	-0.2	100.0	100.0	100.0	100.0	26.6	34.3
		500	20	-44.6	-40.9	-36.5	-36.8	0.1	0.1	100.0	100.0	100.0	100.0	28.8	33.2
		1000	20	-44.8	-41.0	-36.6	-36.8	-0.1	-0.1	100.0	100.0	100.0	100.0	26.7	32.9
	2	500	10	-41.4	-29.0	-25.9	-26.6	-0.1	-0.1	100.0	100.0	100.0	100.0	4.0	5.1
		1000	10	-41.4	-28.9	-25.8	-26.6	0.0	0.1	100.0	100.0	100.0	100.0	6.1	3.9
		500	20	-41.4	-29.2	-25.8	-26.7	0.0	-0.1	100.0	100.0	100.0	100.0	5.3	5.5
		1000	20	-41.4	-29.1	-25.8	-26.6	0.0	0.1	100.0	100.0	100.0	100.0	5.0	5.2

Notes: The value  $\delta$  refers to the fraction of truncated observations,  $\hat{\gamma}_{ls}$  and  $\hat{\beta}_{ls}$  refer to the LS estimates, and  $\hat{\gamma}_{ml}$  and  $\hat{\beta}_{ml}$  refer to the poisson ML estimates. Case 1 refers to the data generating process with  $\sigma_{\epsilon}^2 = 1$ , while case 2 refers to the data generating process with  $\text{var}(M_{ijt}|Y_{ijt}, D_{ijt}) = E(M_{ijt}|Y_{ijt}, D_{ijt})$ . The reported bias results refer to the mean bias times 100.

<sup>a</sup>The LS estimator is based on truncating the sample.

<sup>b</sup>The LS estimator uses  $\ln(M_{ijt} + 1)$  as the dependent variable

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Table 2: Simulation results for the bootstrapped ML  $t$ -test.

$N$	$T$	Case 1				Case 2			
		$t(\widehat{\gamma}_{ml})$	$t(\widehat{\beta}_{ml})$	$t^*(\widehat{\gamma}_{ml})$	$t^*(\widehat{\beta}_{ml})$	$t(\widehat{\gamma}_{ml})$	$t(\widehat{\beta}_{ml})$	$t^*(\widehat{\gamma}_{ml})$	$t^*(\widehat{\beta}_{ml})$
Size at the 5% level									
500	10	23.8	33.4	9.2	10.0	5.8	4.0	10.2	7.4
1000	10	25.2	31.4	10.4	9.6	4.6	5.8	10.0	7.2
500	20	28.4	33.2	7.6	9.6	5.0	6.6	7.8	10.0
1000	20	26.4	39.4	8.4	10.6	6.4	4.8	8.2	6.6
Mean									
500	10	0.0	-0.1	0.0	-0.1	0.1	0.0	0.1	0.1
1000	10	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
500	20	0.0	-0.1	0.0	0.0	0.1	0.0	0.1	0.0
1000	20	0.1	0.0	0.1	0.0	0.0	-0.1	0.0	-0.1
Standard deviation									
500	10	1.7	2.1	1.2	1.2	1.0	1.0	1.2	1.1
1000	10	1.8	1.9	1.2	1.2	1.0	1.0	1.2	1.1
500	20	1.8	2.1	1.1	1.2	1.0	1.1	1.1	1.1
1000	20	1.8	2.2	1.1	1.2	1.0	1.0	1.1	1.1

Notes: The values  $t(\hat{\gamma}_{ml})$  and  $t(\hat{\beta}_{ml})$  refer to the conventional asymptotic ML  $t$ -tests, while  $t^*(\hat{\gamma}_{ml})$  and  $t^*(\hat{\beta}_{ml})$  refer to their bootstrapped counterparts. See Table 1 for an explanation of the remaining features of the table.

Table 3: Empirical estimation results.

Estimator	LS	LS	poisson ML
Dependent variable	$\ln(M_{ijt})$	$\ln(M_{ijt} + 1)$	$M_{ijt}$
$\beta_1$	-0.091 (0.191)	0.229*** (0.062)	0.931*** (0.173)
$\beta_2$	1.438*** (0.084)	0.820*** (0.039)	1.483*** (0.110)
$\beta_3$	4.055*** (0.612)	1.765*** (0.267)	2.471*** (0.629)
$\beta_4$	-1.275*** (0.190)	-0.979*** (0.074)	-0.580 (0.357)
$\gamma_1$	-0.403*** (0.046)	-0.211*** (0.016)	-0.232*** (0.074)
$\gamma_2$	0.000 (0.032)	0.102*** (0.023)	0.041 (0.047)
$\gamma_3$	-0.002 (0.025)	0.033* (0.018)	0.035 (0.034)
No. of country-pairs	2719	2748	2719
No. of observations	32487	35600	35256

*Notes:* The numbers within the parantheses are the robust LS standard errors or the bootstrapped poisson ML standard errors. The superscripts (\*\*), (\*) and (\*\*) denote significance at the 1%, 5% and 10% levels, respectively.

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